Birth of a Closed Universe of Negative Spatial Curvature

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Abstract

We propose a modified form of the spontanteous birth of the universe by quantum tunneling. It proceeds through topology change and inflation, to eventually become a universe with closed spatial sections of negative spatial curvature and nontrivial global topology.

1 INTRODUCTION

The idea of spontaneous birth of the universe by quantum tunneling from a de Sitter instanton has been coupled by Vilenkin [1] to the beginning of the inflationary expansion in a de Sitter spacetime with space sections of positive curvature. We propose an extension of this process, through topology change and inflation, to arrive at a Friedmann model with closed hyperbolic spaces

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(CHS) as spatial sections. We are motivated by previous work by Gott[2] and Bucher et al. [3] in the matter of inflation leading to an $\Omega_0 < 1$ universe, but assume a mechanism different from theirs, relying on De Lorenci et al.'s [4] formalism to quantify the probability of changes in cosmic global topology. There have been suspicions of incompatibility of CHS universes with maps of the cosmic microwave background (CMB)[5]; we refer to a recent paper by Cornish et al. [6] to answer to this criticism.

2 CREATION AND EVOLUTION

We use Planck units $c = G = \hbar = 1$.

De Sitter instanton has the metric (cf. [1])

$$ds^2 = dt'^2 + r_0^2 \cos^2(t'/r_0)(d\chi^2 + \sin^2\chi\ d\Omega^2), \tag{1}$$

where $t' = -it$, with t imaginary time, $d\Omega^2 = d\theta^2 + \sin^2\theta\ d\varphi^2$, (χ, θ, φ) are spherical coordinates, and r_0 is a constant of the order of Planck's length or time. But instead of being defined on a four-sphere S^4, our gravitational instanton has a more general topology: note that (i) the S^4 instanton is obtained by analytic continuation into imaginary time of de Sitter spacetime with topology $R \times S^3$, where the real line R is the time axis and the three-sphere S^3 is the spatial section [7]; (ii) de Sitter's metric of positive spatial curvature can be assigned to any topology of the form $R \times M$, where $M = S^3/\Gamma$ is the quotient space of S^3 by a discrete group of isometries Γ, which acts on S^3 without fixed points - see, for example, Ellis [8], or Lachièze-Rey and Luminet [9]; and (iii) the same process of analytic continuation into

imaginary time can be done on these new spacetime topologies. So we assume as a generalization of de Sitter's instanton the metric of Eq. 1 defined on the quotient spaces S^4/Γ (which may not be manifolds but rather orbifolds - see [10], §2). The volume of M, with its curvature normalized to unity, is $2\pi^2/(\text{order of } \Gamma)$; its fundamental group $\pi_1(M)$ is isomorphic to Γ, hence, except for the case $M = S^3$ (trivial Γ), M is multiply connected.

When this instanton becomes a real de Sitter spacetime, we get the inflationary metric of positive spatial curvature (cf. [1], [2]), starting at $t = 0$,

$$ds^2 = -dt^2 + r_0 \cosh^2(t/r_0)(d\chi^2 + \sin^2\chi \, d\Omega^2), \tag{2}$$

and the topology of the direct product $R \times M$, as implied above. The parameter r_0 is related in [2] to a constant energy density, which is now interpreted as the initial value of the inflaton potential $V(\phi)$. As in [3], during this epoch the inflaton field ϕ is stuck in a false vacuum (region A in Fig. 1), causing a growth that erases inhomogeneities in M. But in our case this stage is very short. In the numerical example of next Section, $t_f = r_0$ is the endtime of this epoch, which therefore may hardly be called inflationary; see below.

At $t = t_f$ we assume that a topology and metric transition takes place; the latter becomes

$$ds^2 = -d\tau^2 + r_0^2 \sinh^2(\tau/r_0)(d\chi'^2 + \sinh^2\chi' d\Omega^2), \tag{3}$$

with initial $\tau = \tau_i$ to be determined below. This is similar to the metric change in [2], but here we shall assume it to happen as a quantum transition in minisuperspace. The change in metric in a closed space implies a change in topology, since the spherical space S^3/Γ cannot support the hyperbolic

metric in the spatial part of Eq. 3 – cf. [10], Theorem 5.2. This metric and topology change is signaled by the small bump in the potential $V(\phi)$ – region B in Fig. 1; we hope a mechanism for this process can be adapted from the one recently developed by De Lorenci et al. [4], who estimated the probabilities of a few similar transitions. Their results, interpreted with some liberty, indicate that a spherical cosmology is unstable against quantum transitions, and has a good chance of becoming hyperbolic, as in our case.

Here the spatial section becomes the CHS $M' = H^3/\Gamma'$, where H^3 is hyperbolic three-space, and $\Gamma' \cong \pi_1(M')$ is a discrete group of isometries acting on H^3 without fixed points – cf. [9]. Spacetime topology becomes $R \times M'$. Since this is a quantum process, we need not demand continuity in the five-dimensional pseudoeuclidean space where de Sitter spacetime is imbedded, as done in [2]. But, as a working hypothesis, we postulate that energy, hence physical volume, is conserved in the transition, and we expect the change in the expansion factor to be small; therefore the normalized volumes of M and M' should be of the same order of magnitude. This can be arranged, and τ_i calculated, as will be seen in the next Section.

When space becomes M' it may again have density irregularites. (To see this, imagine a uniform distribution of a thin film of matter over a two-sphere, which suddenly gets a handle and becomes a torus; the film takes a time to spread itself evenly over the new surface.) In the light of this, it is not so essential that the previous stage gets homogenized, as it is that its duration be short so that M will not grow too much and M' may become smooth; see next Section. At this point one might wonder why is stage M necessary at all. Perhaps because there is no gravitational instanton [11] that would

actualize directly as a closed hyperbolic universe.

Thus M' is neither equivalent to the nucleated bubble in [2] or [3], nor to the smooth patch in Kolb and Turner's [12] basic picture of inflation, for either of these is already homogeneous as it appears on the scene. Besides, M' need not grow to encompass the observable universe; when it does not, as in the example below, the model predicts observable effects of the non-trivial topology. The most obvious of these, but still difficult to verify, is the production of multiple images of each source, so as to mimic the uniform distribution of an open Friedmann model. See [13];[9] and references therein.

In [3] this is the time of slow roll inflation, with $V(\phi)$ sloping as $-\mu^3\phi$ towards the true vacuum. For simplicity here we prefer the potential in [2], which has a plateau (region C in Fig. 1) of about the same height as in the false vacuum, so that inflation proceeds at the same rate as before. This epoch is $\tau_i \leq \tau \leq \tau_1$, with τ_1 being determined by a continuity condition with the next phase.

Finally [2] a phase transition is arranged, the equation of state going from $p = -\rho$ to $p = \rho/3$, which ends the inflationary period. There is reheating around the true vacuum (D in Fig. 1), and spacetime gets the usual Friedmann metric of negative spatial curvature

$$ds^2 = -d\tau^2 + a^2(\tau)(d\chi'^2 + \sinh^2\chi' d\Omega^2). \tag{4}$$

Thus we reach the epoch of standard cosmology, except for the effects of the compactness and multiple connectedness of the spatial sections – see [9].

3 NUMBERS

A convenient family of candidates for $M = S^3/\Gamma$ are the lens spaces $L(p,q)$, where p,q are coprime integers with $1 \leq q \leq p/2$ [14]. Their fundamental group, hence also Γ, is of order p, so their volume is $2\pi^2/p$. Hyperbolic manifolds M' are known to exist with normalized volumes from 0.94 up [15].

As an example, let us take $M = L(50,1)$, with normalized volume $v_{sph} = 0.394784$, and as M' the smallest known CHS, Weeks-Matveev-Fomenko manifold (cf. [16]), with $v_{hyp} = 0.942707$. Then the universe's largest half-diameter, $R_{\max}(t) = (\pi/4)r_0 \cosh(t/r_0)$, is smaller than the radius of the event horizon, $R_H(t) = 2r_0 \cosh(t/r_0)\{\tan^{-1}[\exp(t/r_0)] - \pi/4\}$ for $t > 0.8814r_0$. Let us take $t_f = r_0$ as the endtime of this epoch, which is enough to homogenize M. By the conservation of physical volume, $v_{sph} \cosh^3(1) = v_{hyp} \sinh^3(\tau_i/r_0)$, whence $\tau_i = 0.9865r_0$.

To find τ_1, first we have to make sure that there is enough time for inflation to smoothen an initial inhomogeneity in M'. The circumscribing radius of the Dirichlet domain given in [15] is $r_{\max} = 0.752470$ in comoving (or normalized) units; the maximal distance between points in M' is of this order of magnitude. The horizon's radius is $r_H(\tau) = \ln[\tanh(\tau/2r_0)/\tanh(\tau_i/2r_0)]$. We can have $r_H(\tau) \gtrsim r_{\max}$ only if $\tanh(\tau_i/2r_0) < \exp(-r_{\max})$, or $\tau_i < 1.0232r_0$. With τ_i as obtained above, we must have $\tau_1 > 4.17r_0$. This value is compatible with the one we now obtain from the continuity of the expansion factor, $r_0 \sinh(\tau_1/r_0) = a(\tau_1)$. In the beginning of the Friedmann radiation era that follows reheating, $a(\tau) = (2b_*\tau)^{1/2}$ (cf. [17]), where $b_* = (8\pi G\rho_{rad,0}/3c^2)^{1/2}a_0^2$, with $\rho_{rad,0}$ = present density of radiation energy $= 4.6477 \times 10^{-34}$ g cm^{-3} [12], and a_0 = present value of $a(\tau)$. Taking

$r_0 = 1.6160 \times 10^{-33}$ cm (Planck's length), $\Omega_0 = 0.3$, and Hubble's constant $H_0 = 65$ km s^{-1}Mpc^{-1}, the continuity condition gives $\tau_1 = 71.1 r_0$, close to the value $69 r_0$ in [2].

Today's physical volume of M' would be 4.64×10^{84} cm^3, while the observable universe – interpreted as the region of repeated cosmic images – is about 200 times larger.

4 REMARKS

In the above reasoning the dynamics of spontaneous birth and inflation processes in the modified scenario was presumed to be adaptable from previous results. We plan to elaborate on this point, as also on the matter of topology changes, and to present a more detailed picture in the future. Thermodynamics considerations, prevalent in [2] and not touched upon here, may also be addressed in the wider study.

The question of the influence of space closure on the generation and growth of primordial fluctuations has been left out, to be examined elsewhere. Here we only comment on doubts that have appeared on whether a closed hyperbolic universe would be compatible with the long wavelength modes of the CMB – see [5]. As pointed out in [6], in hyperbolic universes there is no low cutoff for these modes. The confusion seems to arise from mistaking the separations between equivalent points in the space of images H^3 with maximal wavelengths of perturbations; but the latter can be spread over closed geodesics, which may form knotted patterns of increasing length inside the CHS. See also [18], [19]. Therefore these models are probably

compatible with the spotted maps of the CMB obtained by NASA's COBE satellite, but of course more research is needed in this direction.

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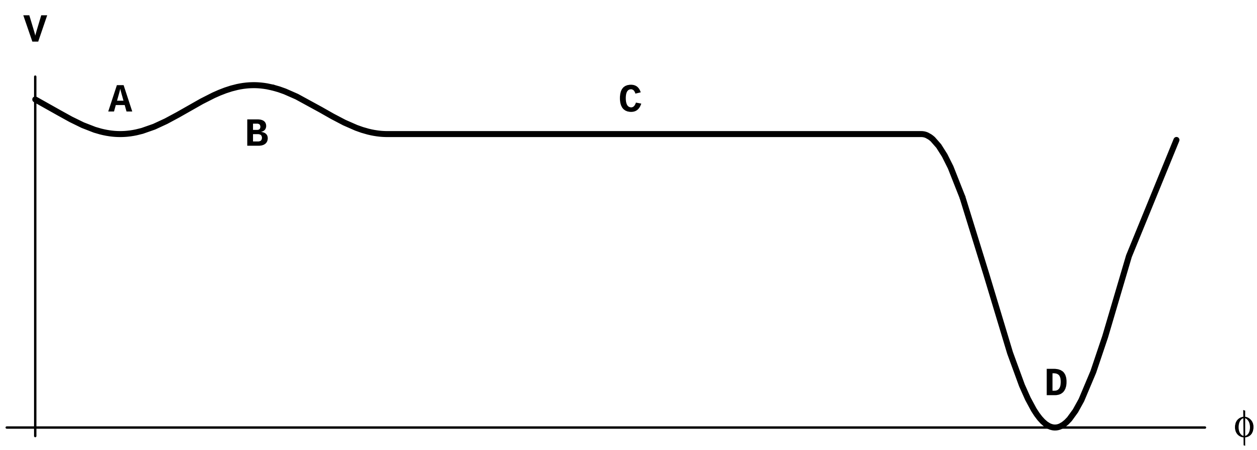
Figure caption:

Fig. 1. This is a qualitative plot of the potential $V(\phi)$. A is the region of transient spherical topology, B is where the tunneling to hyperbolic space happens, C is the region of most inflation, and D is where reheating takes place around the true vacuum.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